

# Kelsenian Jurisprudence, Legal Ontologies and Intuitionistic Logic

Edward Hermann Haeusler

Departamento de Informatica - PUC-Rio - Brasil

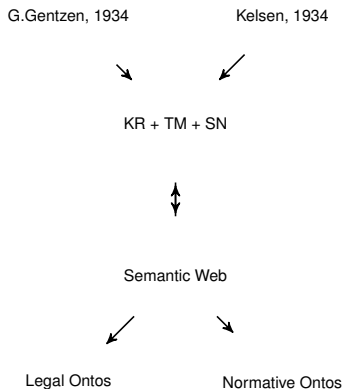
A brief report on the results of a joint work with A. Rademaker (IBM-Research-BR) and V. de Paiva (Univ.Birmingham-UK)

LOIAT2010, JURIX2010, DALI2011, EBL2011, LIX2011, EBL2014

September, 2014



# Historical Scenario



# Purpose of this talk

Remind us how Logic is as important as OntoLogic in Knowledge Representation in IS



# Considerations on Legal Ontologies

## What is an Ontology ?

- A declarative description of a domain.
- Ontology consistency is mandatory.
- Consistency means absence of contradictions.
- Negation is an essential operator.
- Concretely, an Ontology is a Knowledge Base:
- A set of Logical Assertions that aims to describe a Domain completely.



# Considerations on Legal Ontologies

## A T-Box on Family Relationships

<i>Woman</i>	$\equiv$	<i>Person</i> $\sqcap$ <i>Female</i>
<i>Man</i>	$\equiv$	<i>Person</i> $\sqcap$ $\neg$ <i>Woman</i>
<i>Mother</i>	$\equiv$	<i>Woman</i> $\sqcap$ $\exists$ <i>hasChild</i> . <i>Person</i>
<i>Father</i>	$\equiv$	<i>Man</i> $\sqcap$ $\exists$ <i>hasChild</i> . <i>Person</i>
<i>Parent</i>	$\equiv$	<i>Father</i> $\sqcap$ <i>Mother</i>
<i>Grandmother</i>	$\equiv$	<i>Mother</i> $\sqcap$ $\exists$ <i>hasChild</i> . <i>Parent</i>
<i>MotherWithoutDaughter</i>	$\equiv$	<i>Mother</i> $\sqcap$ $\forall$ <i>hasChild</i> . $\neg$ <i>Woman</i>
( $\star$ ) <i>MotherinTrouble</i>	$\equiv$	<i>Mother</i> $\sqcap$ $\geq 10$ <i>hasChild</i>

# Considerations on Legal Ontologies

## What does it mean the term “Law” ?

- What does count as the “unit of law” ? Open question, a.k.a. “The individuation problem”.
- (Raz1972) What is to count as one “complete law”: Naturally justified law versus Positive Law.



# Considerations on Legal Ontologies

## Two main (distinct) approaches to the “Individuation problem”.

- 1 Taking all valid statements as in conformance with a declarative statement of an ideal Legally perfect world. This totality is called “the law”.
  - 2 Taking into account all individually legal valid statement as individual laws positively stated and “The law” is this set.
    - ▷ Facilitates the analysis of structural relationship between laws, viz. Primary and Secondary Rules and explicit Grundnorms.
- ▷ **The second seems to be quite adequate to Legal AI.**



# Considerations on Legal Ontologies

## Why we do not consider Deontic Modal Logic ?

- Deontic Logic does not properly distinguish between the normative status of a situation from the normative status of a norm (rule). (Valente1995)
- Norms should not have truth-value, they are not propositions. (General Theory of Norms, Kelsen 1979/1991, posthumously published)





# Basic Motivations

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable.
- iALC was designed to logically support reasoning on Legal Ontologies based on Kelsen jurisprudence.
- Default iALC is the non-monotonic extension of iALC to deal with the dynamics of legal processes.



# Our approach: the (static) part of a trial

- Considering a jurisprudence basis, classical **ALC** is not adequate to our approach. We use an intuitionistic version, **iALC**.
- Dealing with the common (deontic) paradoxes.
- A proof-theoretical basis to legal reasoning and explanation.
- **laws** are inhabitants of a universe that must be formalized.
- Propositions are about laws and not the laws themselves.

Haeusler, De Paiva, Rademaker (2010-2011-2013/14).



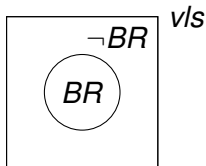
# Formalization of a Legal System

- The first-class citizens of any Legal System are vls. Only vls inhabit the legal world.
- There can be concepts (collections of laws) on vls and relationships between vls. For example: *PIL<sub>BR</sub>*, *CIVIL*, *FAMILY*, etc, can be concepts. *LexDomicilium* can be a relationship, a.k.a. a legal connection.
- The relationships between concepts facilitates the analysis of structural relationships between laws.
- The natural precedence between laws, e.g. “Peter is liable” precedes “Peter has a renting contract”, is modeled as a special relationships between laws.



# Intuitionistic versus Classical logic

- The extension of an **ALC** concept is a **Set**.



- Classical Negation:  $\neg\phi \vee \phi$  is valid for any  $\phi$ .

In  $BR$ , 18 is the legal age

$BR$  contains all v/s in Brazil

“Peter is 17”

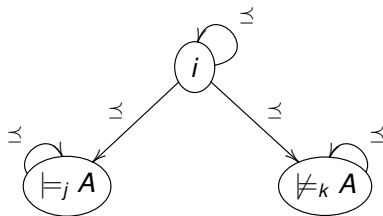
“Peter is liable”  $\notin BR$  iff “Peter is liable”  $\in \neg BR$

- Classical negation forces the “Peter is liable” be valid in some legal system outside Brazil.



## Intuitionistic versus Classical logic (cont.)

- The Intuitionistic Negation  $\models_i \neg A$ , iff, for all  $j$ , if  $i \preceq j$  then  $\not\models_j A$



$$\not\models_i \neg\neg A \rightarrow A \quad \text{and} \quad \not\models_i A \vee \neg A$$

- In an intuitionistically based approach to Law, we can have neither “Peter is liable”  $\notin BR$  nor “Peter is liable”  $\in \neg BR$ .
- $pl \in \neg BR$  means  $pl : \neg BR$  means  $\mathcal{I}, pl \models \neg BR$  or  $\forall z. z \succeq pl$  we have  $z \not\models BR$ .
- In other words, there is no  $z$  with  $z \succeq pl$  such that  $\mathcal{I}, z \models BR$ . There is no vls in  $BR$  dominating “Peter is liable”.



# A logic for legal theories formalization

- Binary (Roles) and unary (Concepts) predicate symbols,  $R(x, y)$  and  $C(y)$ .
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.

$$C, D ::= A \mid \perp \mid \top \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid C \sqsubseteq D \mid \exists R.C \mid \forall R.C$$

$A$  are general assertions and  $N$  nominal assertions for ABOX reasoning. Formulas ( $F$ ) also includes subsumption of concepts interpreted as propositional statements.

$$N ::= x : C \mid x : N \quad A ::= N \mid xRy \mid x \leq y \quad F ::= A \mid C \sqsubseteq D$$

where  $x$  and  $y$  are nominals,  $R$  is a role symbol and  $C, D$  are concepts.



# A Sequent Calculus for iALC

$$\begin{array}{c}
 \frac{}{\Delta, \delta \Rightarrow \delta} \\
 \frac{\Delta, xRy \Rightarrow y: \alpha}{\Delta \Rightarrow x: \forall R. \alpha} \forall\text{-r} \\
 \frac{\Delta \Rightarrow xRy \quad \Delta \Rightarrow y: \alpha}{\Delta \Rightarrow x: \exists R. \alpha} \exists\text{-r} \\
 \frac{\Delta, \alpha \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqsubseteq \beta} \sqsubseteq\text{-r} \\
 \frac{\Delta \Rightarrow \alpha \quad \Delta \Rightarrow \beta}{\Delta \Rightarrow \alpha \sqcap \beta} \sqcap\text{-r} \\
 \frac{\Delta \Rightarrow \alpha}{\Delta \Rightarrow \alpha \sqcup \beta} \sqcup\text{-r} \\
 \frac{\Delta, \alpha \Rightarrow \beta}{\forall R. \Delta, \exists R. \alpha \Rightarrow \exists R. \beta} p\text{-}\exists \\
 \frac{\Delta \Rightarrow \delta}{x: \Delta \Rightarrow x: \delta} p\text{-N}
 \end{array}$$

$$\begin{array}{c}
 \frac{}{\Delta, x: \perp \Rightarrow \delta} \\
 \frac{\Delta, x: \forall R. \alpha, y: \alpha, xRy \Rightarrow \delta}{\Delta, x: \forall R. \alpha, xRy \Rightarrow \delta} \forall\text{-l} \\
 \frac{\Delta, xRy, y: \alpha \Rightarrow \delta}{\Delta, x: \exists R. \alpha \Rightarrow \delta} \exists\text{-l} \\
 \frac{\Delta_1 \Rightarrow \alpha \quad \Delta_2, \beta \Rightarrow \delta}{\Delta_1, \Delta_2, \alpha \sqsubseteq \beta \Rightarrow \delta} \sqsubseteq\text{-l} \\
 \frac{\Delta, \alpha, \beta \Rightarrow \delta}{\Delta, \alpha \sqcap \beta \Rightarrow \delta} \sqcap\text{-l} \\
 \frac{\Delta, \alpha \Rightarrow \delta \quad \Delta, \beta \Rightarrow \delta}{\Delta, \alpha \sqcup \beta \Rightarrow \delta} \sqcup\text{-l} \\
 \frac{\Delta \Rightarrow \alpha}{\forall R. \Delta \Rightarrow \forall R. \alpha} p\text{-}\forall
 \end{array}$$

All propositional rules have their nominal version.



# Using **iALC** to formalize Conflict of Laws in Space

*Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 21. Peter lives in Edinburgh and Maria lives in Rio.*

Only legally capable individuals have civil obligations:

*PeterLiab*  $\preceq$  *ContractHolds@RioCourt*, shortly, *pl*  $\preceq$  *cmp*

*MariaLiab*  $\preceq$  *ContractHolds@RioCourt*, shortly, *ml*  $\preceq$  *cmp*

Concepts, nominals and their relationships:

*BR* is the collection of Brazilian Valid Legal Statements

*SC* is the collection of Scottish Valid Legal Statements

*PIL<sub>BR</sub>* is the collection of Private International Laws in Brazil

*ABROAD* is the collection of VLS outside Brazil

*LexDomicilium* is a legal connection: the pair  $\langle pl, pl \rangle$  is in *LexDomicilium*





# Non-Logical Axiom Sequents

The sets  $\Delta$ , of concepts, and  $\Omega$ , of **iALC** sequents representing the knowledge about the case.

$$\Delta = \boxed{\begin{array}{l} ml : BR \quad pl : SC \quad pl \preceq cmp \\ ml \preceq cmp \quad pl \text{ LexDom } pl \end{array}}$$

$$\Omega = \boxed{\begin{array}{l} \text{PIL}_{BR} \Rightarrow BR \\ SC \Rightarrow \text{ABROAD} \\ \exists \text{LexD}_1.L_1 \dots \sqcup \exists \text{LexDom.ABROAD} \sqcup \dots \sqcup \exists \text{LexD}_k.L_k \Rightarrow \text{PIL}_{BR} \end{array}}$$



# A proof in our SC

$$\begin{array}{c}
 \frac{\Delta \Rightarrow pl : SC \quad \frac{\Omega}{pl : SC \Rightarrow pl : A}}{\Delta \Rightarrow pl : A} \text{cut} \quad \frac{\Delta \Rightarrow pl \text{ LexD } pl}{\Delta \Rightarrow pl : \exists \text{LexD}.A} \exists\text{-R} \\
 \frac{\frac{\frac{\frac{\exists \text{LexD}.A \Rightarrow \exists \text{LexD}.A}{\exists \text{LexD}.A \Rightarrow \text{PIL}_{BR}}{\text{PIL}_{BR} \Rightarrow BR} \sqcup\text{-R}}{\exists \text{LexD}.A \Rightarrow BR} \text{p-N}}{pl : \exists \text{LexD}.A \Rightarrow pl : BR} \text{cut}}{\Delta \Rightarrow pl : BR} \text{cut} \\
 \\
 \frac{\Delta \Rightarrow ml : BR \quad \frac{\frac{\frac{\Pi}{\Delta \Rightarrow pl : BR} \quad \frac{\frac{\Omega}{ml : BR, pl : BR \Rightarrow cmp : BR}}{\Delta, ml : BR \Rightarrow cmp : BR} \text{cut}}{\Delta \Rightarrow cmp : BR} \text{cut}}{\Delta \Rightarrow cmp : BR} \text{cut}
 \end{array}$$



# Comparing with the deontic logic approach

## Considerations on the logical nature of laws

- 1 Deontic approach: **Laws** must be taken as **propositions** ?, or
- 2 iALC/Kelsenian approach: **Laws** are inhabitants of a universe that must be formalized, i.e:

*Main question: Propositions are about laws ? or they are the laws themselves ?*



# Comparing with the deontic logic approach

## Contrary-to-duty paradoxes

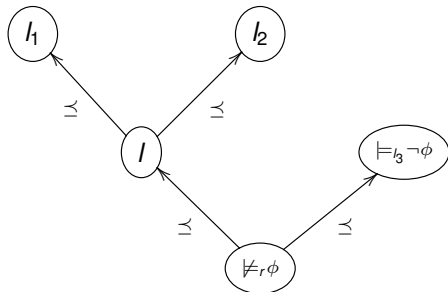
It ought to be that Jones goes to assist his neighbors.	$Ob(\phi)$
It ought to be that if Jones goes, then he tells them he is coming.	$Ob(\phi \rightarrow \psi)$
If Jones doesn't go, then he ought not tell them he is coming.	$\neg\phi \rightarrow Ob(\neg\psi)$
Jones doesn't go.	$\neg\phi$

$\phi$  is “Jones goes to assist his neighbors”

$\psi$  is “Jones tells his neighbors he is coming”

## An *iALC* model for the Chisholm (ex) paradox

- 1 The law  $I_1$ , originally  $Ob(\phi)$ ;
- 2 The law  $I_2$ , originally  $Ob(\phi \rightarrow \psi)$ ;
- 3 The law  $I_3$ , orig.  $\neg\psi$ , and the assertion " $I_3 : \neg\phi$ ", orig.  $\neg\phi \rightarrow Ob(\neg\psi)$ ;
- 4 A concept  $\neg\phi$ ;
- 5 The law  $I$  that represents the infimum of  $I_1$  and  $I_3$



## Metatheorems

- *iALC* is sound and complete regarded Intuitionistic Conceptual Models (Hylo 2010)
- $IPL \subset iALC$  (hardness is PSPACE)
- Alternating Polynomial Turing-Machine to find out winner-strategy on the SAT-Game of a hybrid language. (upper-bound is PSPACE).



# Conclusions

- It is fully adequate to (at least one) jurisprudence theory.
- Juridic cases can be analyzed with the help of ABOX (assertions on particular laws).
- TBOX describes “The Law”.
- $\preceq$  is not always specified at the level of the TBOX.
- It seems to scale, but there is no empirical evidence.
- (?) Work out “hard juridical cases”.
- (?) Can be the kernel of a tool for helping with a judge's decision (not a sentence writer!!!)



# THANK YOU

