Intuitionistic Description Logic and Legal Reasoning

Edward Hermann Hausler
Valeria de Paiva
Alexandre Rademaker

Departamento de Informática - PUC-Rio - Brasil
FGV - Brasil
Univ. Birmingham - UK

DALI 2011 august

Basic Motivation

Some facts

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable (TBOX PSPACE, TBOX+ABOX EXPTIME).
- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes);
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes;
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Basic Motivation

Some facts

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable (TBOX PSPACE, TBOX+ABOX EXPTIME).
- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes);
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes;
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Basic Motivation

Some facts

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable (TBOX PSPACE, TBOX+ABOX EXPTIME).
- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes);
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes;
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Basic Motivation

Some facts

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable (TBOX PSPACE, TBOX+ABOX EXPTIME).
- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes);
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes;
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Basic Motivation

Some facts

- Description Logic is among the best logical frameworks to represent knowledge.
- Powerful language expression and decidable (TBOX PSPACE, TBOX+ABOX EXPTIME).
- Deontic logic approach to legal knowledge representation brings us paradoxes (contrary-to-duty paradoxes);
- *ALC*, as a basic *DL*, might be considered to legal knowledge representation if it can deal with the paradoxes;
- Considering a jurisprudence basis, classical *ALC* it is not adequate to our approach.

Basic Motivation

Our approach

- An intuitionistic version of $\mathcal{ALC}$ tailored to represent legal knowledge.
- PSPACE complexity of $i\mathcal{ALC}$.
- Dealing with the paradoxes.
- A proof-theoretical basis to legal reasoning and explanation.

Formalizing a Legal System

A fundamental question in jurisprudence:

- What does count as the "unit of law" ? Open question, a.k.a. "The individuation problem".
- (Raz1972) What is to count as one "complete law" ?

Formalizing a Legal System

What is the purpose of "the law"

- Legal positivism tradition (Kelsen1934/1991): "The law" rules the society.
- An immediate the question shows up: "How does one maintain "law coherence"?
  1. Is it Naturally obtained ? Is it regarded to describe an ideal (natural) world ??, or;
  2. Is it resulted from a Knowledge Management process on smaller legal parts ??

Formalizing a Legal System

Two possible formal attitudes to take into account:

1. Taking the collection of laws as a whole. A law, or general law, is a kind of deontic statement or proposition.
2. Taking into account all individual legal valid statements (ivls or vls for short) as individual laws. An individual law is not a deontic statement, it is not even a proposition.

Considerations on the logical nature of laws

- **laws** must be taken as **propositions** ?, or
- **laws** are inhabitants of a universe that must be formalized, i.e:
- Propositions are about laws ? or they are the laws themselves ?

Contrary-to-duty paradoxes

It ought to be that Jones go to the assistance of his neighbours.	$Ob(\phi)$
It ought to be that if Jones does go then he tells them he is coming.	$Ob(\phi \to \psi)$
If Jones doesn't go, then he ought not tell them he is coming.	$\neg\phi \to Ob(\neg\psi)$
Jones doesn't go.	$\neg\phi$

ϕ is "Jones go to the assistance of his neighbours"

ψ is "Jones tells his neighbours he is coming"

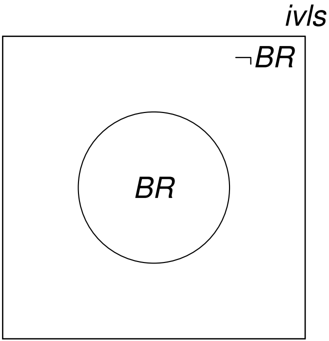
Formalization of a Legal System following the second approach

- The first-class citizens of any Legal System are vls. Only vls inhabit the (legal).
- There can be concepts on vls and relationships between vls. For example: PIL_{BR}, *CIVIL*, *FAMILY*, etc, can be concepts. *LexDomicilium* can be a relationship, a.k.a. a legal connection.
- Facilitates the analysis of structural relationships between laws, viz. Primary and Secondary Rules. Induces natural precedence between laws, e.g. " Peter is liable" precedes "Peter has a renting contract".

Intuitionistic versus Classical logic:

Which version is more adequate to Law Formalization??

The extension of an *ALC* a concept is a **Set**



Intuitionistic versus Classical logic:

Which version is more adequate to Law Formalization??

Classical Negation: $\neg\phi \vee \phi$ is valid for any ϕ

In *BR*, 18 is the legal age   *BR* contains all vls in Brazil

.

" Peter is 17 "

"Peter is liable"$\notin$ *BR* iff "Peter is liable"$\in \neg$*BR*

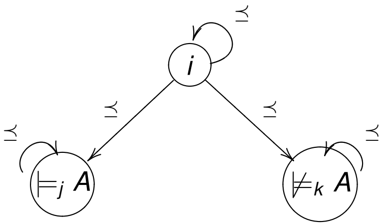
Classical negation forces the "Peter is liable" is valid in some legal system outside Brazil

Intuitionistic versus Classical logic:

Which version is more adequate to Law Formalization??

The Intuitionistic Negation

$\models_i \neg A$, iff, for all j, if $i \preceq j$ then $\not\models_j A$



$$\not\models_i \neg\neg A \rightarrow A \quad \text{and} \quad \not\models_i A \vee \neg A$$

Intuitionistic versus Classical logic:

Which version is more adequate to Law Formalization??

An Intuitionistically based approach to Law

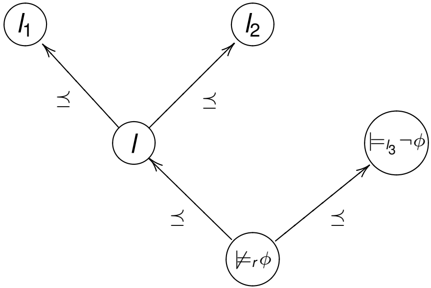
“Peter is liable” $\notin BR$

“Peter is liable” $\in \neg BR$ means | There is no vls in BR dominating “Peter is liable” |

neither “Peter is liable” $\notin BR$ nor “Peter is liable” $\in \neg BR$

An *iALC* model for the Chisholm (ex) paradox

1. The law l1, originally $Ob(\phi)$;
2. The law l2, originally $Ob(\phi \rightarrow \psi)$;
3. The law l3, orig. $\neg\psi$, and the assertion “$l3 : \neg\phi$”, orig. $\phi \rightarrow Ob(\neg\psi)$;
4. A concept $\neg\phi$;
5. The law l that represents the infinum of $l1$ and $l3$



The logical framework for legal theories formalization

iALC and *ALC* have the same logical language

- Binary (Roles) and unary (Concepts) predicate symbols, $R(x, y)$ and $C(y)$.
- Prenex Guarded formulas $(\forall y(R(x, y) \rightarrow C(y))$, $\exists y(R(x, y) \wedge C(y)))$.
- Essentially propositional (Tboxes), but may involve reasoning on individuals (Aboxes), expressed as "$i : C$".
- Semantics: Provided by a structure $\mathcal{I} = (\Delta^{\mathcal{I}}, \preceq^{\mathcal{I}}, \cdot^{\mathcal{I}})$ closed under refinement, i.e., $y \in A^{\mathcal{I}}$ and $x \preceq^{\mathcal{I}} y$ implies $x \in A^{\mathcal{I}}$. "$\neg$" and "$\sqsubseteq$" must be interpreted intuitionistically .
- It is not First-order Intuitionistic Logic. It is a genuine Hybrid logic.

Deductive Reasoning in *iALC*

Usual Structural-Rules for Intuitionistic Logic

$$\frac{\Delta \Rightarrow x : A \qquad A \Rightarrow B}{\Delta \Rightarrow x : B}\ \in\text{-r}$$

$$\frac{}{\Gamma, x\colon C \Rightarrow x\colon C, \Delta}$$

$$\frac{}{xRy, \Gamma \Rightarrow \Delta, xRy}$$

$$\frac{\Gamma_1 \Rightarrow C \qquad \Gamma_2, D \Rightarrow \Delta}{\Gamma_1, \Gamma_2, C \sqsubseteq D \Rightarrow \Delta}\ \sqsubseteq\text{-l}$$

$$\frac{\Gamma, C \Rightarrow D}{\Gamma \Rightarrow C \sqsubseteq D}\ \sqsubseteq\text{-r}$$

$$\frac{\Gamma, x\colon C, x\colon D \Rightarrow \Delta}{\Gamma, x\colon (C \sqcap D) \Rightarrow \Delta}\ \sqcap\text{-l}$$

$$\frac{\Gamma \Rightarrow x\colon C, \Delta \qquad \Gamma \Rightarrow x\colon D, \Delta}{\Gamma \Rightarrow x\colon (C \sqcap D), \Delta}\ \sqcap\text{-r}$$

$$\frac{\Gamma, x\colon C \Rightarrow \Delta \qquad \Gamma, x\colon D \Rightarrow \Delta}{\Gamma, x\colon (C \sqcup D), \Rightarrow \Delta}\ \sqcup\text{-l}$$

$$\frac{\Gamma \Rightarrow x\colon C, x\colon D, \Delta}{\Gamma \Rightarrow x\colon (C \sqcup D), \Delta}$$

$$\frac{\Gamma, x\colon \forall R.C, y\colon C, xRy \Rightarrow \Delta}{\Gamma, x\colon \forall R.C, xRy \Rightarrow \Delta}\ \forall\text{-l}$$

$$\frac{\Gamma, xRy \Rightarrow y\colon C, \Delta}{\Gamma \Rightarrow x\colon \forall R.C, \Delta}\ \forall\text{-r}$$

$$\frac{\Gamma, xRy, y\colon C \Rightarrow \Delta}{\Gamma, x\colon \exists R.C \Rightarrow \Delta}\ \exists\text{-l}$$

$$\frac{\Gamma \Rightarrow \Delta, xRy \qquad \Gamma \Rightarrow \Delta, y\colon C}{\Gamma \Rightarrow \Delta, x\colon \exists R.C}\ \exists\text{-r}$$

Using *iALC* to formalize Conflict of Laws in Space

A Case Study

> *Peter and Maria signed a renting contract. The subject of the contract is an apartment in Rio de Janeiro. The contract states that any dispute will go to court in Rio de Janeiro. Peter is 17 and Maria is 20. Peter lives in Edinburgh and Maria lives in Rio.*

Only legally capable individuals have civil obligations:

$PeterLiable \preceq ContractHolds@RioCourt$, shortly, $pl \preceq cmp$

$MariaLiable \preceq ContractHolds@RioCourt$, shortly, $ml \preceq cmp$

Concepts, nominals and their relationships

BR is the collection of Brazilian Valid Legal Statements
SC is the collection of Scottish Valid Legal Statements
PIL_{BR} is the collection of Private International Laws in Brazil
ABROAD is the collection of *VLS* outside Brazil
LexDomicilium is a legal connection:

Legal Connections The pair $\langle pl, pl \rangle$ is in *LexDomicilium*

Non-Logical Axiom Sequents

The sets Δ, of concepts, and Ω, of *iALC* sequents representing the knowledge about the case

$$\Delta = \boxed{\begin{array}{ccc} ml : \mathrm{BR} & pl : \mathrm{SC} & pl \preceq cmp \\ ml \preceq cmp & pl\ \mathrm{LexDom}\ pl & \end{array}}$$

$$\Omega = \boxed{\begin{array}{c} \mathrm{PIL_{BR}} \Rightarrow \mathrm{BR} \\ \mathrm{SC} \Rightarrow \mathrm{ABROAD} \\ \exists \mathrm{LexD_1.L_1} \ldots \sqcup \exists \mathrm{LexDom.ABROAD} \sqcup \ldots \exists \mathrm{LexD_k.L_k} \Rightarrow \mathrm{PIL_{BR}} \end{array}}$$

In Sequent Calculus

$$\dfrac{\dfrac{\dfrac{\Delta \Rightarrow pl : \mathsf{SC} \qquad \dfrac{\Omega}{pl : \mathsf{SC} \Rightarrow pl : \mathsf{A}}}{\Delta \Rightarrow pl : \mathsf{A}}\,cut \qquad \Delta \Rightarrow pl \ \mathsf{LexD} \ pl}{\Delta \Rightarrow pl : \exists \mathsf{LexD.A}}\,\exists - R \qquad \dfrac{\dfrac{\exists \mathsf{LexD.A} \Rightarrow \exists \mathsf{LexD.A}}{\exists \mathsf{LexD.A} \Rightarrow \mathsf{PIL}_{\mathsf{BR}}}\,\sqcup\text{-R} \qquad \dfrac{\Omega}{\mathsf{PIL}_{\mathsf{BR}} \Rightarrow \mathsf{BR}}}{\exists \mathsf{LexD.A} \Rightarrow \mathsf{BR}}\,cut}{\Delta \Rightarrow pl : \mathsf{BR}}\,inc - R$$

$$\dfrac{\Delta \Rightarrow ml : \mathsf{BR} \qquad \dfrac{\dfrac{\Pi}{\Delta \Rightarrow pl : \mathsf{BR}} \qquad \dfrac{\Omega}{\boxed{ml : \mathsf{BR}, pl : \mathsf{BR} \Rightarrow cmp : \mathsf{BR}}}}{\Delta, ml : \mathsf{BR} \Rightarrow cmp : \mathsf{BR}}\,cut}{\Delta \Rightarrow cmp : \mathsf{BR}}\,cut$$

Metatheorems

- *iALC* is sound and complete regarded Intuitionistic Conceptual Models (Hylo 2010)
- *IPL* $\subset$ *iALC* (hardness is PSPACE)
- Alternating Polynomial Turing-Machine to find out winner-strategy on the SAT-Game of a hybrid language. (upper-bound is PSPACE).

$SAT_{iALC} \subset PSPACE$

- One wants fo verify whether $\Theta, \Gamma \Rightarrow \gamma$ is satisfiable.
- $\Theta, \Gamma \Rightarrow \gamma$ is satisfiable, if and only if, $(\sqcap_{\theta \in \Theta} \theta) \sqsubseteq \gamma$ is satisfiable in a model of Γ. A game is defined on $\Gamma \cup \{\xi\}$
- $\exists$*loise* starts by playing a list $\{H_0, \ldots, H_k\}$ of $\Gamma \cup \{\xi\}$ of Hintikka I-sets, and two relations $\mathcal{R}$ and $\preccurlyeq$ on them.
- $\exists$*loise* loses if she cannot provide the list as a pre-model.
- $\forall$*belard* chooses a set from the list and a formula inside this set.
- $\exists$*loise* has to fulfill extend the (pre)-model in order to satisfy the formula.
- $\Gamma \cup \xi$ is satisfiable, iff, $\exists$*loise* has a winning strategy.

Conclusions

- It is fully adequate to (at leats one) jurisprudence theory.
- Juridic cases can be analyzed with the help of ABOX (assertions on particular laws).
- TBOX describes “The Law”.
- $\preceq$ is not always specified at the level of the TBOX.
- It seems to scale, but there is no empirical evidence.
- (?) Work out “hard juridical cases”.
- (?) Can be the kernel of a tool for helping with a judge’s decision (not a sentence writer!!!)

THANK YOU